

# C. Encrypted Communication

Time limit: 1s

Memory limit: 512MB

The frontline demining fleas have successfully obtained all mine coordinates! Now, the only task left is to transmit this information to headquarters.

To prevent information leakage, the communication must be encrypted. A large prime  $p = 10^9 + 7$  is chosen, and a function  $f : \mathbb{N}^+ \rightarrow \mathbb{Z}_p$  is defined to be **multiplicative** if and only if:

- $f(1) = 1$ , and
- for all  $n, m \geq 1$ , if  $\gcd(n, m) = 1$ , then

$$f(nm) \equiv f(n)f(m) \pmod{p}.$$

The frontline encodes the mine information into two multiplicative functions  $f$  and  $g$ , computes their sum

$$s = (f + g) \pmod{p},$$

and sends  $s$  to headquarters.

As a communication expert flea, you are given the values  $s(1), s(2), \dots, s(n)$ . Your task is to reconstruct a valid pair of multiplicative functions  $f$  and  $g$ .

- If multiple solutions exist, output the **lexicographically smallest**  $f$ .
- If no valid solution exists, output  $-1$ .

A function  $f_1$  is lexicographically smaller than  $f_2$  if there exists an index  $1 \leq i \leq n$  such that:

- for all  $1 \leq j < i$ ,  $f_1(j) = f_2(j)$ , and
- $f_1(i) < f_2(i)$ .

## Input Format

This is a multiple-testcase problem.

The first line contains an integer  $T$ , the number of test cases.

For each test case:

- The first line contains an integer  $n$ .
- The second line contains  $n$  non-negative integers  $s(1), s(2), \dots, s(n)$ .

## Output Format

For each test case, output one line:

- If no solution exists, output  $-1$ .
- Otherwise, output  $n$  integers representing  $f(1), f(2), \dots, f(n)$  for the lexicographically smallest valid solution.

## Sample Input

```
2
6
2 1 4 5 1 4
20
2 12 9 11 18 69 20 9 24 102 39 72 12 72 76 21 31 120 24 90
```

## Sample Output

```
1 0 0 0 0 0
1 3 2 1 10 6 18 0 16 30 0 2 0 54 20 0 0 48 0 10
```

## Additional Samples

Samples 2–4 are provided in the attachments, corresponding to subtasks 2, 5, 6.

## Constraints

For all test cases:

- $n \geq 1$
- $1 \leq T \leq \sum n \leq 10^6$

- $0 \leq s(i) \leq 10^9 + 6$

Subtasks are designed with dependencies.

| Subtask | $\sum n \leq$ | Special Property | Score |
|---------|---------------|------------------|-------|
| 1       | 14            | None             | 10    |
| 2       | 100           | <b>CD</b>        | 10    |
| 3       | 1000          | <b>AD</b>        | 8     |
| 4       | 1000          | <b>BD</b>        | 16    |
| 5       | $10^5$        | <b>A</b>         | 8     |
| 6       | $10^5$        | <b>B</b>         | 16    |
| 7       | $10^6$        | <b>C</b>         | 12    |
| 8       | $10^6$        | None             | 20    |

## Special Properties

- **A:**  $s$  is guaranteed to be the sum of two independently randomly generated valid multiplicative functions (random over prime powers).
- **B:** There exists a valid solution  $(f, g)$  such that for all  $2 \leq i \leq n$ ,  $f(i) \neq g(i)$ .
- **C:** There exists a valid solution  $(f, g)$  where both  $f$  and  $g$  are completely multiplicative functions.
- **D:**  $n \geq 20$ .